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PROBLEMS OF PHYSICAL MODELING OF ELECTRIC-ARC DISCHARGES

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Special features of physical modeling of high-current arc discharges are considered. It is shown that the employment of dimensionless criterial expressions makes it possible to establish only approximate similarity of characteristics for some specific apparatuses which govern arc-discharge conditions. Procedures to select the dominant energy-exchange processes and to find the scale values of plasma properties are presented, whose knowledge permits modeling of the discharges initiated in various media.

1. Introduction. Complexity of many phenomena hinders the use of mathematical modeling methods to determine the characteristics of the occurring processes. In these cases, we are led to resort to experimental data and derive, on their basis, formulas for description of the regularities of the phenomenon. Sometimes, experimental data are put into the form of empirical expressions that demonstrate the interrelation between the function and individual arguments. But the most convenient form to describe the regularities sought is representation of experimental results as dimensionless similarity numbers. The generalized relations that are obtained on their basis make it possible to more completely describe the regularities.

The application of similarity methods to electric-arc discharges, however, is confronted by a number of difficulties due both to the diversity of the processes in the discharge and a wide temperature range. The number of the controlled parameters is usually small (just several quantities) but they act on a multitude of closely interrelated processes of different physical natures—electric, magnetic, gasdynamic, optical, etc., processes. The interrelation between the individual processes is described by similarity numbers, whose number, if only the basic phenomena are allowed for, exceeds significantly the number of varied initial parameters. Therefore, for electric-arc discharges, only approximate similarity methods can be used. In this case, there arises the problem of determining the dominant processes that have the major effect on discharge characteristics. This problem is complicated by the dependence of the basic processes on discharge conditions whose variation can lead to a complete change in the prevailing phenomena. The large temperature difference between the center of the discharge and its periphery is due to a variation in plasma properties over a wide range over the arc column cross-section. But some constant "scale" values of physical properties that would characterize correctly all phenomena as a whole must appear in the similarity numbers. The choice of these values of plasma properties is a difficult problem. Nonetheless, rather convenient methods for determining scale values of plasma properties for electric-arc discharges have been found by now.

Incomplete similarity is associated with additional errors of the derived formulas that are due to neglect of a number of insignificant phenomena. At the same time, there are common problems of choosing approximating expressions that would ensure the smallest deviation from real regularities for statistical processing of experimental data and the subsequent employment of the formulas derived. When the number of generalized arguments is limited it is desirable to determine, along with estimation of the total error, the degree of generalized function stratification by the individual dimensional initial variables that are the components of the dimensionless similarity numbers.

In what follows we consider some methods to overcome the difficulties of physical modeling of arc discharges that are based both on the use of standard regression-analysis methods and on the special approaches developed for arc-discharge conditions.

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2. Preliminary Estimation of the Role of Individual Processes. The role of individual processes can be estimated by comparison, using the characteristic values of dimensional quantities, of the significance levels for different terms of the equation that describe these phenomena. It is convenient to compare the significance of dimensionless criteria. To do this, the estimated equations should be previously reduced to their dimensionless form. It is clear that, with approximate similarity, we can refrain from further consideration of those terms of the equation that prove to be by several orders of magnitude smaller that the dominant ones.

As an example let us consider the equation for the electric current density (its sufficiently complete value is obtained in [1] by the methods of nonequilibrium thermodynamics)

$$\mathbf{j} = \sigma \mathbf{E} + \sigma [\mathbf{v}_c \mathbf{B}] - \sigma b [\mathbf{j}\mathbf{B}] - \frac{\sigma m_i}{e\rho_c} [\mathbf{j}\mathbf{B}] - \sigma \rho_c b \mathbf{g} + \sigma b \rho_c \frac{d\mathbf{v}_c}{dt} -$$

$$- \frac{\sigma m_e m_i}{e^2} \frac{d}{dt}\left(\frac{\mathbf{j}}{\rho_c}\right) + \sigma b \,\mathrm{grad}\, P_c + \frac{\sigma m_i}{2e\rho_c} \mathrm{grad}\, P_c +$$

$$+ \frac{\sigma \kappa_{ei} k}{2e} \mathrm{grad}\, T - \frac{\sigma n_0 \rho_c b k}{2(\rho_0 + \rho_e)} \mathrm{grad}\, T - \frac{\sigma n_0 m_i k}{2(\rho_0 + \rho_e)} \mathrm{grad}\, T. \quad (1)$$

The most important component of the current density is usually due to the external electric field E. Therefore it is appropriate to compare all the remaining components precisely with it. For this purpose, it is desirable to reduce Eq. (1) to dimensionless form by dividing it through by the modulus of the electric field strength E

$$\frac{\mathbf{j}}{\sigma E} = \underset{(1)}{\frac{\mathbf{E}}{E}} + \underset{(2)}{\frac{[\mathbf{v}_c \mathbf{B}]}{E}} - \underset{(3)}{\frac{b[\mathbf{j}\mathbf{B}]}{E}} - \underset{(4)}{\frac{m_i [\mathbf{j}\mathbf{B}]}{e\rho_c E}} - \underset{(5)}{\frac{\rho_c b \mathbf{g}}{E}} + \underset{(6)}{\frac{b\rho_c}{E}\frac{d\bar{\mathbf{v}}_c}{dt}} -$$

$$- \underset{(7)}{\frac{m_i m_e}{e^2 E}\frac{d}{dt}\left(\frac{\mathbf{j}}{\rho_c}\right)} + \underset{(8)}{\frac{b}{E}\mathrm{grad}\, P_c} + \underset{(9)}{\frac{m_i}{2e\rho_c E}\mathrm{grad}\, P_c} + \underset{(10)}{\frac{\kappa_{ei} k}{2eE}\mathrm{grad}\, T} -$$

$$- \underset{(11)}{\frac{n_0 \rho_c b k}{2(\rho_0 + \rho_e) E}\mathrm{grad}\, T} - \underset{(12)}{\frac{n_0 m_i k}{2(\rho_0 + \rho_e) eE}\mathrm{grad}\, T}. \quad (2)$$

The relative role of the individual terms of the equation will be governed by the discharge conditions. For example, magnetic induction-dependent terms of the equation will not be essential to an unmagnetized thermal plasma. With a stationary d.c. discharge, we can disregard the effect of a variation in the velocity of charged particles and current density with time while the terms that are associated with a pressure gradient can be important if the arc is initiated in a supersonic nozzle. The temperature gradient can have a pronounced effect on the periphery of the arc column but the arc characteristics as a whole are not markedly affected by this.

The relative significance of the individual components of the current density can be roughly estimated by choosing some numerical values of each of the quantities that are characteristic of these discharge conditions. For example, for a stationary arc of industrial-frequency alternating current in an air flow with $M = 0.5$ at atmospheric pressure and $I = 1000$ A, we take $d = 10^{-2}$ m and $l = 10^{-1}$ m. We consider $T_0 = 9 \cdot 10^3$ K as the characteristic temperature, according to [2], at the interface of the central and peripheral regions of the arc column. From this temperature, we determine the physical properties of the plasma [3-5] (Table 1). The data of Table 1 enable us to estimate the individual terms of Eq. (2) (see Table 2).

TABLE 1. Characteristic Values of Quantities for an Alternating-Current Air Arc

Quantity and dimensions	Value	Source
T, K	9000	Taken
I, A	1000	ditto
v_c, m·sec^{-1}	1260	ditto M = 0
d, m	10^{-2}	ditto
l, m	10^{-1}	ditto
b, T	0.5	ditto
t, sec	10^{-2}	ditto
j, A·m^{-2}	$1.27 \cdot 10^{7}$	Calculated
E, V·m^{-1}	4421	ditto
B, m^3·A^{-1}·sec^{-1}	$-7.9 \cdot 10^{-6}$	ditto [1]
α_{ei}, dimensionless	1.41	ditto [1]
σ, Ω^{-1}·m^{-1}	2880	ditto [4]
P_c, Pa	840	ditto [3]
n_c, m^{-3}	$6.8 \cdot 10^{21}$	ditto [3]
n_0, m^{-3}	$8 \cdot 10^{23}$	ditto [3]
m_i, kg	$2.44 \cdot 10^{-26}$	ditto [3]
m_e, kg	$9.106 \cdot 10^{-31}$	ditto [3]
ρ_c, kg·m^{-3}	$1.66 \cdot 10^{-4}$	ditto [3]
ρ_0, kg·m^{-3}	$1.98 \cdot 10^{-2}$	ditto [3]
ρ_e,kg·m^{-3}	$6.2 \cdot 10^{-9}$	ditto [3]
k, J·K^{-1}	$1.38 \cdot 10^{-1}$	ditto [5]
e, kl	$1.6 \cdot 10^{-19}$	ditto [5]
g, m·sec^{-2}	9.807	ditto [5]

TABLE 2. Estimation of the Relative Values of Terms of Eq. (2)

No. of term of Eq. (2)	1	2	3	4	5	6
Estimation	1	$1.43 \cdot 10^{-1}$	$1.13 \cdot 10^{-2}$	1.32	$2.91 \cdot 10^{-12}$	$3.74 \cdot 10^{-8}$
No. of term of Eq. (2)	7	8	9	10	11	12
Estimation	$1.5 \cdot 10^{-11}$	$1.5 \cdot 10^{-5}$	$8.73 \cdot 10^{-4}$	$2.48 \cdot 10^{-2}$	$1.48 \cdot 10^{-4}$	$1.73 \cdot 10^{-2}$

As the data of Table 2 show, for magnetic–induction values of about fractions of a tesla which is sometimes used in electric-arc plasma generators for rotary displacement of arc attachments over the surface of cold electrodes to keep them from burning through, even at atmospheric pressure it can prove to be necessary to allow for the terms that contain magnetic induction. The role of gravitational forces is negligibly small. In arcs of industrial-frequency alternating current, the terms that allow for nonstationary processes are also unimportant. Nonstationary processes should, apparently, be taken into account in the case of pulsed discharges or for high frequencies. When the pressure differences are small, which is the case with subsonic flows, we can disregard the terms with pressure gradients. The temperature gradients were averaged over the adopted values of the temperature and arc column diameter. In fact, they have a much greater effect on the arc's periphery. Therefore, in some cases, it is appropriate to take into account radial currents that are due to thermal-diffusion processes.

3. Determination of Scale Values of Plasma Properties. Knowledge of the scale values of plasma properties is required with employment of generalized formulas to calculate the characteristics of arc discharges that are initiated in media of various compositions, for example, in heating of different gases or as the composition of the heated mixture varies. However, the problem of determining the characteristic scales is strongly hindered by a large temperature difference within the arc column and the dependence of local temperatures on discharge conditions. If the arc is initiated in a certain medium, the indicated difficulties can be avoided by employing "incomplete" similarity numbers in which there are no values of plasma properties. In these numbers, similarity in all other variables except physical properties is retained. One usually acts precisely in this manner when obtaining generalized characteristics of the arc in dimensional form for individual gases. When generalized formulas are employed to calculate the characteristics of an arc that is initiated in various media, in [2], it is proposed to approximate the dependence $\sigma(h)$ by two straight-line segments. The values of σ_0, h_0, and T_0 are determined at the point of the straight lines' intersection. The temperature thus obtained corresponds to the region between the central part and periphery of the arc column.

In the method considered, the dependence of T_0 on the character of the dominant heat-exchange process is disregarded. This analysis is made in [6] for weakly stabilized arc discharges. In the central part of unstable discharges, temperature profiles are rather flat in character. Therefore a characteristic scale value of the temperature must approximately correspond to the arc-column temperature, since integral volt-ampere characteristics are governed mainly by the electrical conductivity of its central part. However, this characteristic value of the temperature (we denote it as T^*) depends on the arc-burning regime, whose change acts on the temperature in the arc column. The method for determining a constant scale value of the temperature T_0 is based on allowance for this feature of a variation in T^* with discharge conditions.

If the discharge characteristics can to sufficient accuracy be described by one dominant criterion, for example, by the number of convective heat exchange $\pi_{\rm conv}^{\rm long} = Gd\sigma_0 h_0/I^2$, the generalized equation will contain one dimensionless argument. When used as a function of the generalized resistance $f = Ud\sigma_0/I$ the VAC of the discharge becomes

$$\frac{Ud\sigma_0}{I} = C\left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{\alpha}. \tag{3}$$

When a variable, burning regime-dependent, characteristic value of the temperature T_0 is taken instead of its constant scale value T_*, the number of independent variables reduces by unity. Accordingly the number of independent similarity number decreases. Then the function assumes its constant value $Ud\sigma^*/I$ = const while the argument with "drifting" properties is equal to unity

$$\frac{Gd\sigma^* h^*}{I^2} = 1\,. \tag{4}$$

This equality is applicable to determination of the scale values of plasma properties from their temperature dependence. It is convenient to use a power approximation of the properties in the form: $\sigma^*/\sigma_0 = (T^*/T_0)^n$, $h^*/h_0 = (T^*/T_0)^m$, etc. A very narrow temperature range can be taken for the approximation so that $T^* \approx T_0 \approx T$. In this case, the exponents $n(T)$ and $m(T)$ are determined from the slope of a tangent line of the functions $\sigma(T)$ and $h(T)$ while the approximation error is minimized even for spike-like functions of the $\lambda(T)$ type.

When the indicated power approximations are used for a temperature dependence of plasma properties we can write

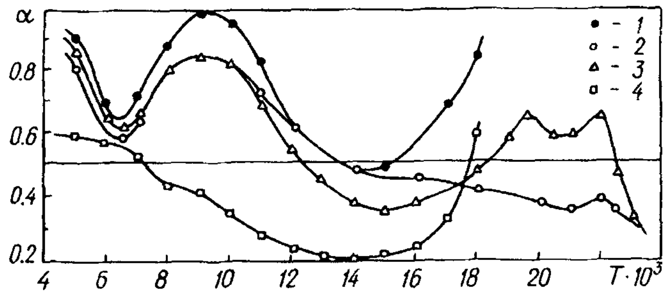

Fig. 1. Temperature dependence of the exponents for one-criterion models of generalized VACs of an arc discharge in air: 1, 2, 3, and 4) dependences for the similarity numbers of energy transfer by thermal turbulence, heat conduction, convection, and radiation, respectively.

$$\frac{Gd\sigma_0 h_0}{I^2}\left(\frac{T^*}{T_0}\right)^{n+m} = 1 \tag{5}$$

instead of (4). This yields

$$\frac{T^*}{T_0} = \left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{-\frac{1}{n+m}}, \tag{6}$$

while the generalized VAC assumes the form

$$\frac{Ud\sigma_0}{I} = C\left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{\frac{n}{n+m}}. \tag{7}$$

Thus, it turns out that in the case of one generalized argument its exponent can be determined theoretically from the known temperature dependences of plasma properties and the scale value of the temperature T_0. Conversely, from the experimental magnitude of the exponent $\alpha = n/(n+m)$ we can find T_0. To that corresponds the investigated parameter range for an electric-arc apparatus. If it is taken into account that the VAC of a high-current weakly stabilized arc is approximately parallel to the current axis, for this arc in the case of dominant convective heat transfer $\alpha \approx 0.5$, since the current appears in the generalized function to a power of -1 while its exponent in the generalized argument is equal to -2. Therefore, for the weakly stabilized arc, the scale value of T_0 can to sufficient accuracy be found theoretically by the temperature dependence of plasma properties. In the general case for other modes of heat transfer, if the current appears in the generalized function to a power of -1, the exponent of the argument for $U = \text{const}$ is $\alpha = 1/p$, where p is the exponent of the power to which the current appears in the generalized argument. For example, the criterion of convective heat transfer for a magnetomovable arc has the form $\pi_{\text{conv}}^{\text{magn}} = \rho_0\sigma_0^2 h_0^2 BL^5/I^3$. In this case, $p = -3$ and, to ensure parallelism of VAC to the current axis ($U = \text{const}$), α must be equal to 1/3.

Figure 1 exemplifies determination of the scale value of the temperature for a weakly stabilized arc in an air atmosphere for $P = 10^5$ Pa. The exponents α for the generalized arguments that characterize different modes of heat transfer in a transversely blown arc depend here on temperature: $\pi_{\text{conv}}^{\text{long}} = Gd\sigma_0 h_0/I^2$; $\pi_{\text{cond}} = \sigma_0\lambda_0 T_0 d^2/I^2$; $\pi_{\text{turb}} = \rho_0\sigma_0 h_0^{1.5} d^3/I^3$; $\pi_{\text{rad}} = Q\sigma d^4/I^2$. All these numbers involve the current to a power of -2. Therefore, the scale values of the temperature are determined when the curves intersect with the horizontal line $\alpha = 0.5$. For π_{conv}, π_{cond}, and π_{turb}, they are similar and lie within $T_0 = 12{,}300$–$13{,}300$ K. For radiant heat transfer, $T_0 = 7300$ K.

If the discharge characteristics are governed by several processes, the number of generalized arguments should be increased accordingly. The fraction of participation for a process χ_i will be defined by the ratio of the exponent on the corresponding criterion α_i to the value of the exponent that it assumes with sole domination. For a

TABLE 3. Scale Values of the Temperature and Plasma Properties of Some Gases for Different Dominant Heat-Transfer Processes

Dominant mechanism of heat transfer	Quantity and dimensions	Scale values			
		air	argon	hydrogen	helium
Convection, longitudinal blowing, $\alpha = 0.5$	T_0, K	12,300	10,700	12,300	17,500
	σ_0, $\Omega^{-1}\cdot m^{-1}$	5630	3438	4558	5820
	h_0, $J\cdot kg^{-1}$	$6.62\cdot 10^{7}$	$7.44\cdot 10^{6}$	$6.70\cdot 10^{8}$	$1.55\cdot 10^{8}$
	η_0, $kg\cdot m^{-1}\cdot sec^{-1}$	$2.10\cdot 10^{-4}$	$2.32\cdot 10^{-4}$	$6.60\cdot 10^{-5}$	$3.76\cdot 10^{-4}$
	λ_0, $W\cdot m^{-1}\cdot K^{-1}$	2.03	0.75	5.32	6.80
	ρ_0, $kg\cdot m^{-3}$	$1.26\cdot 10^{-2}$	$4.37\cdot 10^{-2}$	$9.00\cdot 10^{-4}$	$2.55\cdot 10^{-3}$
Heat conduction, $\alpha = 0.5$	T_0, K	13,400	10,300	15,700	19,000
	σ_0, $\Omega^{-1}\cdot m^{-1}$	6800	3042	8014	7920
	h_0, $J\cdot kg^{-1}$	$8.28\cdot 10^{7}$	$6.63\cdot 10^{6}$	$1.45\cdot 10^{9}$	$2.20\cdot 10^{8}$
	η_0, $kg\cdot m^{-1}\cdot sec^{-1}$	$1.68\cdot 10^{-4}$	$2.40\cdot 10^{-4}$	$1.67\cdot 10^{-5}$	$2.72\cdot 10^{-4}$
	λ_0, $W\cdot m^{-1}\cdot K^{-1}$	2.40	0.64	6.44	8.55
	ρ_0, $kg\cdot m^{-3}$	$1.05\cdot 10^{-2}$	$4.60\cdot 10^{-2}$	$6.28\cdot 10^{-4}$	$2.17\cdot 10^{-3}$
Thermal turbulence, $\alpha = 0.5$	T_0, K	13,400	10,700	12,900	17,500
	σ_0, $\Omega^{-1}\cdot m^{-1}$	6800	3438	5270	5820
	h_0, $J\cdot kg^{-1}$	$8.28\cdot 10^{7}$	$7.44\cdot 10^{6}$	$7.61\cdot 10^{8}$	$1.55\cdot 10^{8}$
	η_0, $kg\cdot m^{-1}\cdot sec^{-1}$	$1.68\cdot 10^{-4}$	$2.38\cdot 10^{-4}$	$5.50\cdot 10^{-5}$	$3.76\cdot 10^{-4}$
	λ_0, $W\cdot m^{-1}\cdot K^{-1}$	2.40	0.75	5.82	6.80
	ρ_0, $kg\cdot m^{-3}$	$1.05\cdot 10^{-2}$	$4.37\cdot 10^{-2}$	$8.16\cdot 10^{-4}$	$2.55\cdot 10^{-3}$
Convection (magnetomovable arc), $\alpha = 0.333$	T_0, K	11,800	9800	11,800	16,600
	σ_0, $\Omega^{-1}\cdot m^{-1}$	5060	2538	3978	4606
	h_0, $J\cdot kg^{-1}$	$6.04\cdot 10^{7}$	$5.85\cdot 10^{6}$	$6.00\cdot 10^{8}$	$1.30\cdot 10^{8}$
	η_0, $kg\cdot m^{-1}\cdot sec^{-1}$	1.80	0.53	4.88	5.95
	λ_0, $W\cdot m^{-1}\cdot K^{-1}$	$2.37\cdot 10^{-4}$	$2.88\cdot 10^{-4}$	$7.43\cdot 10^{-5}$	$4.14\cdot 10^{-4}$
	ρ_0, $kg\cdot m^{-3}$	$1.36\cdot 10^{-2}$	$4.88\cdot 10^{-2}$	$9.57\cdot 10^{-4}$	$2.77\cdot 10^{-3}$

weakly stabilized arc ($U = \text{const}$), this ratio has the form $\chi_i = -p_i\alpha_i$, and since $\sum_i \chi_i = 1$, $\sum_i (-p_i\alpha_i) = 1$; and if p_i is the same for all the numbers, $\chi_i = -p_i\alpha_i/\sum_i (-p_i\alpha_i) = \alpha_i/\sum_i \alpha_i$. As the discharge regime changes the role of the individual processes can vary. This must reflect on the exponents on the individual generalized arguments.

Table 3 gives the scale values of the temperature and plasma properties for some conditions of heat transfer in arc burning in various media.

It is pertinent to note that, since $U = \text{const}$ corresponds to the minimum voltage, i.e., the region of VAC transition from the descending branch to the ascending one, T_0 that is determined at this point can be employed

TABLE 4. Analysis of Power Approximation Adequacy for Description of VAC of an Arc Vortex Plasmatron

Parameter		Flow rate of gas (air) G, g/sec								
		1.27	1.64	2.05	2.45	2.57	2.94	3.83	4.27	Entire set
$F_{in.ad}$		1.60	1.68	1.63	2.01	31.0	1.31	5.93	13.3	13.7
F_{regr}		738	1114	1046	1958	709	1579	981	1018	9829
F_{tab}	5%	2.26	2.23	2.28	2.32	2.28	2.28	2.26	2.23	1.31
	1%	3.17	3.12	3.21	3.29	3.21	3.21	3.17	3.12	1.48

for both branches as the scale value. Therefore, the presented procedure for finding the scale values of plasma properties can be applied not only to flat VACs.

4. Determination of Dominant Processes and Obtaining Generalized VACs. The degree of the influence of an argument in the regression equation can be estimated quantitatively by the standardized coefficients and by the partial value of the Fisher variance ratios. Each of the arguments takes over a part of the sum of the squares of deviations of the experimental values of the function from the grand average. The remaining part is the sum of the squares of deviations from regression ("remainders"). The coefficients of the regression equation are determined by the least squares method with the chosen approximation method for the regularity sought. The variance that is determined from this sum of the squares of deviations and referred to the variance of the remainders yields the value of the Fisher parameter. If the derived value proves to be smaller than the tabulated one that corresponds to the degrees of freedom for both variances, the contribution of this argument does not fall outside the limits of a random spread, and we can disregard it. The available standard programs of regession analysis make it possible to determine the partial values of the Fisher parameter for each argument and, from these data, to select the dominant criteria by a step method.

An important step of regression analysis is the choice of a suitable expression by which we can approximate to sufficient accuracy the regularity sought. The simplest and most convenient is a linear regression, to which the expressions with linear dependences in logarithmic and semilogarithmic coordinates reduce. Approximations in the form of power dependences have gained the greatest acceptance.

We checked the possibility of using power functions to approximate the arc-discharge VAC [7]. The error that was introduced by approximation was numerically estimated by the adequacy parameter, which is the ratio of the variance of the spread caused by the approximation inaccuracy to the variance of the deviation from regression due to random factors (a "net error"). To distinguish the approximation error from the total spread, we performed, on a vortex air plasmatron, special experiments with a repetition of each experimental point of the VAC of up to 6 times. From the spread in the values of the function the sum of the squares of deviations for a net error was determined in repeat experiments. Subtraction of this sum from the total spread with respect to regression yields the value of the sum of the squares of deviations due to approximation inadequacy. Approximations of the form $U = CI^{\alpha}$ for individual gas flow rates and $U = CI^{\alpha}G^{\beta}$ for all the flow rates combined in a plasmatron with $d = 1.0$ cm, $I = 38-146$ A, and $G = 1.26-4.27$ g/sec were analyzed.

In Table 4, the obtained values of the Fisher inadequacy parameter $F_{in.ad}$ are compared with the corresponding tabulated values. It can be seen that the experimental values of $F_{in.ad}$ are similar to the tabulated ones; for some flow rates, they are smaller, and for others, somewhat larger than the tabulated data. For comparison, the table also gives the Fisher parameters for regression. The result obtained shows that the power approximation describes the arc-discharge VAC sufficiently well.

Certain difficulties also emerge in the step of selecting the dominant processes. Since the similarity numbers are grouped from dimensional variables, the same quantities are parts of different generalized arguments. Considerable difficulties emerge that are associated with ensuring the independence of the generalized arguments of one another. Accordingly, estimation of the degree of their partial effect on discharge characteristics by the methods of regression analysis ("multicollinearity") is complicated.

TABLE 5. Regression Parameters for the Generalized VAC $\ln (u\alpha\sigma_0/I) = \ln c + \alpha \ln \pi_{\text{turb}} + \pi \ln \pi_{\text{Hall}}$ for Large Electrode Gaps

$R-SQ$	Standard error SE	Regression coefficient			Coefficient of correlation between π_{turb} and π_{Hall}	Partial values of F_{regr}	
		$\ln C$	α	β		π_{turb}	π_{Hall}
0.995	0.055	−0.847	0.556	0.152	−0.0195	27,140	785

Let us consider the effect of discharge conditions using a magnetomovable arc as an example. An electric arc that moves along an electrode under the action of a magnetic field is a very unstable object, since the only stabilizing factor is the electrode material jets issuing from reference points of the arc through electrode evaporation under the action of intense heating. The stabilizing action of these jets is attenuated as the electrode gap increases, the oscillations of discharge parameters increasing. For these conditions, the processes of convective and turbulent energy exchange are dominant. The latter must grow in importance with an increasing electrode gap.

However, some other phenomena, too, can have a certain effect. Therefore, to select by the step method, the following independent variables $\pi_{\text{conv}}^{\text{magn}} = \rho_0\sigma_0^2h_0^2/L^5B/I^3$; $\pi_{\text{turb}} = \rho_0\sigma_0h_0^{1.5}L^3/I^2$; $\pi_{\text{cond}} = \sigma_0\lambda_0T_0L^2/I^2$; $\pi_{\text{rad}} = \sigma_0Q_{r0}L^4/I^2$; $\text{Re} = (IBL\rho_0/\eta_0^2)^{0.5}$; $\pi_{\text{ind}} = \sigma_0^2L^3B^3/\rho_0I$; $\pi_{\text{Hall}} = \sigma_0B/en_{e0}$. were included as components of the objects tested. The Reynolds number Re must ensure allowance for the effect of gasdynamic turbulence, unlike π_{turb}, which describes the transfer of energy by thermal turbulence. The numbers π_{conv}, π_{cond}, and π_{rad} allow for energy transfer by convection, conduction, and radiation, respectively, while π_{ind} and π_{Hall} reflect the presence of the induced electromotive force and the Hall effect. The generalized resistance is used as a dependent variable (function): $f = UL\sigma_0/I$.

Table 5 gives regression parameters for the VAC of a high-current arc that moves in air along parallel electrodes in an external magnetic field ("railtron"). Experimental data are borrowed from [8]. The discharge parameters were varied within the limits: I = 100−1000 A, B = 0.012−0.108 T, and L = 12.7−38.00 mm. The designations of the pairs of electrode gaps correspond to the magnitudes: 1-2 (12.7−19.1 mm); 2-3 (19.1−25.4) mm; 3-4 (25.4−32.0) mm; 4-5 (32.0−38.0) mm. As the analysis shows (see Table 1 at page 547 of this issue of the journal) convective energy transfer has a dominant role only for small gaps $L \leq 19.1$ mm but then it gives way to thermal turbulence, which is supplemented by the Hall effect. Conduction is outside the important quantities while π_{rad} and Re have negative regression parameters and must be excluded from consideration, too. The same is also true for π_{conv} of the third pair of the gaps.

Thus, step selection of the dominant processes showed that an increase in the electrode gap $L > 19.1$ mm leads to a change in the dominant character of the energy exchange from a convective character to a thermal-turbulence one. Characteristically, the Reynolds number, which allows for the influence of energy transfer through gasdynamic turbulence, fell outside the number of the dominant ones. This can be explained by the different levels of heat energy and kinetic energy. For the temperature $T = 10^4$ K, which is characteristic of the arc, and the number M = 1, the air plasma enthalpy exceeds by an order of magnitude the specific kinetic energy of the flow. For subsonic velocities, this relation can attain several orders of magnitude. Therefore, the primary gasdynamic instability excites thermal disturbances, whose energy effect proves to be much stronger.

It is also of interest that the Hall effect proves to be a factor at atmospheric pressure and relatively small values of magnetic induction.

5. **Estimation of Similarity by Individual Dimensional Parameters.** Standard regression-analysis programs generate data on the resultant root-mean-square deviation of the function from the regression equation. This deviation includes both the spread due to random factors and stratification by some dimensional parameters that appear in the dimensionless arguments, since the similarity is approximate. In a number of cases, however, it becomes necessary to estimate the degree of stratification by these variables. To do this, we can use a procedure

TABLE 6. Regression Parameters for VAC of a Railtron with Different Electrode Gaps. Correlation of the Form $\ln f = \ln C + \alpha \ln \pi_{turb} + \beta \ln \pi_{Hall}$; $L \geq 19.1$ mm

Size of gap	SS_i	N	S_i^2	$F_{regr.i}$
19.1	0.0872713	32	0.00272723	2616
25.4	0.0930713	28	0.00332397	3020
32.0	0.0388778	31	0.00125414	5842
38.0	0.0858771	39	0.00220198	2540

that is similar to the method for estimating the inadequacy of the approximating expression. The Fisher parameter for stratification can be determined from the ratio of the stratification variance to the variance of the random spread

$$F_{strat} = S_{strat}^2 / S_{rand}^2 . \tag{8}$$

If F_{strat} proves to be smaller that the tabulated Fisher parameters for the prescribed degrees of freedom, the stratification does not fall outside the limits of the random spread, and we can disregard it.

The sum of the squares of deviations due to stratification SS_{strat} can be defined as the difference of the sum of the squares of deviations from the general curve SS_{gen} and the sum of the squares of deviation due to random stratification SS_{rand}. To calculate the latter, we can determine the regression parameters for the analyzed generalized characteristic at some partial values of the dimensional variable under consideration. If approximation inadequacy is included in the random spread, the sum of the squares of deviation from a partial regression equation will represent a partial value of the sum for a net error. Then $SS_{rand} = \sum_i SS_i$. In a similar way, we determine the number of the degrees of freedom $N_{rand} = \sum_i N_i$. In this case, $S_{rand}^2 = SS_{rand}/N_{rand} = \sum_i SS_i / \sum_i N_i$; $SS_{strat} = SS_{gen} - \sum_i SS_i$; $N_{strat} = N_{gen} - \sum_i N_i$ and $S_{strat}^2 = SS_{strat}/N_{strat}$.

We present, as an example, the estimation of the stratification of the generalized VAC of the above-considered railtron by partial values of the gap for a VAC of the form $f = C\pi_{turb}^{\alpha}\pi_{Hall}^{\beta}$ (data to calculate F_{strat} are given in Table 6): $SS_{gen} = 0.423679$; $SS_{rand} = \sum SS_i = 0.3050975$. Accordingly $SS_{strat} = SS_{gen} - SS_{rand} = 0.1185375$; $N_{rand} = 130$; $N_{strat} = 9$; $S_{rand}^2 = SS_{rand}/N_{rand} = 2.3469038 \cdot 10^{-3}$; $S_{strat}^2 = SS_{strat}/N_{strat} = 1.3170166 \cdot 10^{-2}$. Hence $F_{strat} = 5.6$.

The tabulated values are: $F_{tab} = 2.01$ (5%) and 2.55 (1%). Thus, stratification by gap falls somewhat outside the limits of the random spread. Stratification by magnetic induction is not observed in the case considered.

Analysis of the Remainders. Estimation of the relative role of individual arguments of the regression equation yields correct results when the assumptions that were made to develop the analysis procedureare observed. It is necessary that the remainders have a zero average, obey the normal law with the same variance, and be independent. The investigation methods for the remainders are usually included in standard regression-analysis programs. The analysis of the remainders for the VAC of arc discharges shows that approximate similarity leads, in some cases, to a substantial stratification by individual parameters, which can induce a deviation from a normal distribution. A strong correlation between individual generalized variables has an effect on the mutual correlation of the remainders, too. Methods to allow for deviations from ideality have been developed [9, 10] that should be used to obtain the correct result in estimating the relative role of the dominant processes of approximate similarity.

7. Conclusions. The presence in an electric arc of numerous interrelated processes that occur at high temperatures and large temperature gradients hinders substantially theoretical calculations to determine discharge characteristics. Therefore, we are led to resort to the methods of physical modeling using experimental data. The employment of dimensionless numbers makes it possible to establish only approximate similarity of the characteristics for some discharge conditions on which the prevailing influence of certain mechanisms of heat transfer depends.

It is appropriate to determine the dominant mechanisms, to select the most important similarity numbers, and to derive expressions for discharge characteristics by statistical methods with standard programs of regression analysis. Some experience in physical modeling is currently being accumulated, which makes it possible to obtain generalized characteristics for arcs initiated in various media, which reduces substantially the volume of the necessary experimental data and accelerates the development of electric-arc apparatuses.

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NOTATION

B, magnetic induction; b, coefficient that allows for friction between plasma components; d, diameter; E, electric field strength; e, electron charge; F, variance ratio; f, function; G, gas flow rate; g, free fall acceleration; h, enthalpy; I, current; j, current density; k, Boltzmann constant; L, characteristic dimension; l, distance; M, Mach number; m, mass of particle; N, number of degrees of freedom; n, concertration of particles in unit volume; P, pressure; p, exponent; Q, emittance; $R-SQ$, determination coefficient; SS, sum of the squares of deviations from regression; S^2, variance; SE, standard regression error; T, temperature; t, time; U, voltage; v, velocity; κ, thermal-diffusion coefficient; α and β, exponents; η, viscosity; λ, thermal conductivity; π, similarity number; ρ, density; σ, electrical conductivity; χ, degree of influence of process. Subscripts: c, referred to charged particles; e, electronic; i, ionic; 0, neutral, scale value; conv, convective; cond, conductive; turb, turbulent; rad, radiant; ind, induced; Hall, Hall effect; strat, stratification; rand, random; gen, general; regr, reegression; in.d, inadequate; long, longitudinal arc blowing; magn, magnetomovable arc.

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